



**Calculator Assumed  
Applications of Logarithms**

Time: 45 minutes  
Total Marks: 45  
Your Score: / 45

**Question One: [3, 3, 3 = 9 marks]**

**CA**

The magnitude of an earthquake, known as the Richter scale, is given by:

$$M = \log_{10} I - \log_{10} S$$

where  $M$  is the magnitude of the earthquake,  $I$  is the intensity measurement of the earthquake and  $S$  is the standard intensity of an earthquake.

- (a) The magnitude of the 2011 Christchurch earthquake measured 6.3 on the Richter scale. Find an expression for the intensity of this earthquake in terms of  $S$ .
- (b) If in the same year there was another earthquake that was three times stronger in intensity, what was the magnitude of this second earthquake?

## Mathematics Methods Unit 4

- (c) In 2013 an earthquake in the Solomon Islands measured 8.0 on the Richter scale. How much more intense was this earthquake compared with the one in Christchurch?

**Question Two: [1, 2, 2, 2 = 7 marks] CA**

The mass  $M$ , in grams, of a radioactive substance after  $t$  years is given by :

$$M = 13.8 - \ln(t + 43.1)$$

- (a) State the initial mass of the substance.
- (b) Determine an expression for the instantaneous rate of change of the mass with respect to time.
- (c) Calculate the decrease in mass in the 100<sup>th</sup> year.
- (d) Calculate the average decrease in the mass over 100 years.

**Question Three: [2, 2, 3, 2, 3 =12 marks] CA**

Kepler's Third Law states that the square of the orbit of the period of a planet,  $T$ , is proportional to the cube of its average distance from the sun,  $a$ .

(a) State Kepler's Third Law in terms of  $T$  and  $a$ .

(b) The table below gives values of  $a$  and  $T$ , using Earth units.

Planet	Earth	Mercury	Mars	Jupiter	Saturn
$a$	1	0.3870	1.523	5.203	9.539
$T$	1	0.2410	1.881	11.86	29.46
$\ln a$					
$\ln T$					

Calculate the rows for  $\ln a$  and  $\ln T$ .

(c) Using your CAS calculator, fit a linear regression model using the data from the table. Input the values for  $\ln a$  in List 1 and the values for  $\ln T$  in List 2. State the linear regression model.

(d) By stating the value of the correlation coefficient,  $r$ , explain why a linear model is appropriate.

## Mathematics Methods Unit 4

Since the vertical intercept of the answer to (c) is almost 0, we can say that  $\ln T \approx 1.5 \ln a$

- (e) Hence show using algebraic manipulation that Kepler's Third Law (as stated in part (a) ) holds true from our data.

### **Question Four: [2, 2, 2, 3 = 9 marks] CA**

Laura is starting a new fitness routine and she completes 2 sets of 5 repetitions of squats each day. Her aim is to get stronger and lift heavier each day.

Laura models her progress over  $t$  days by the function:  $f(t) = 5 + k \ln(t + 10)$  , where  $f(t)$  is the weight in kilograms of her squat each day.

- (a) Calculate the value of  $k$  if initially Laura lifts 30 kg.
- (b) After 2 weeks of training, by how many kilograms has her strength increased?
- (c) Calculate the rate of change of Laura's strength with respect to time.
- (d) Determine when Laura's increase in strength is half of what it was initially.

**Question Five: [3, 2, 2 =7 marks] CA**

The instantaneous rate of change of the number of fish over  $t$  weeks, being farmed in a fish farm can be modelled by  $P'(t) = \frac{-4970}{t+e}$  where  $P(t)$  is the population after  $t$  weeks.

- (a) If after 5 weeks there are 12 000 fish left, determine an expression for  $P(t)$ .
- (b) Calculate the initial number of fish when the study began.
- (c) When the decline in fish each week falls below 500, the farmer is no longer as concerned for his fish stock. During which week does this occur?



**SOLUTIONS**  
**Calculator Assumed**  
**Applications of Logarithms**

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**Question One: [3, 3, 3 = 9 marks]**

**CA**

The magnitude of an earthquake, known as the Richter scale, is given by:

$$M = \log_{10} I - \log_{10} S$$

where  $M$  is the magnitude of the earthquake,  $I$  is the intensity measurement of the earthquake and  $S$  is the standard intensity of an earthquake.

- (a) The magnitude of the 2011 Christchurch earthquake measured 6.3 on the Richter scale. Find an expression for the intensity of this earthquake in terms of  $S$ .

$$6.3 = \log_{10} I - \log_{10} S \quad \checkmark$$

$$6.3 = \log_{10} \frac{I}{S} \quad \checkmark$$

$$10^{6.3} = \frac{I}{S}$$

$$I = 10^{6.3} \times S \quad \checkmark$$

- (b) If in the same year there was another earthquake that was three times stronger in intensity, what was the magnitude of this second earthquake?

$$M = \log_{10} \frac{3I}{S} \quad \checkmark$$

$$M = \log_{10} 3I - \log_{10} S$$

$$M = \log_{10} 3 + \log_{10} I - \log_{10} S \quad \checkmark$$

$$M = \log_{10} 3 + \log_{10} \frac{I}{S}$$

$$M = \log_{10} 3 + 6.3$$

$$M = 6.78 \quad \checkmark$$

## Mathematics Methods Unit 4

- (c) In 2013 an earthquake in the Solomon Islands measured 8.0 on the Richter scale. How much more intense was this earthquake compared with the one in Christchurch?

$$8.0 = \log_{10} I - \log_{10} S$$

$$8.0 = \log_{10} \frac{I}{S} \quad \checkmark$$

$$10^8 = \frac{I}{S}$$

$$I = 10^8 \times S \quad \checkmark$$

$$\frac{I_{\text{Solomon}}}{I_{\text{Christchurch}}} = \frac{10^8 \times S}{10^{6.3} \times S} = 10^{1.7} \approx 50 \quad \checkmark$$

Approximately 50 times more intense.

### Question Two: [1, 2, 2, 2 = 7 marks] CA

The mass  $M$ , in grams, of a radioactive substance after  $t$  years is given by:

$$M = 13.8 - \ln(t + 43.1)$$

- (a) State the initial mass of the substance.

$$M = 13.8 - \ln 43.1 = 10.04g \quad \checkmark$$

- (b) Determine an expression for the instantaneous rate of change of the mass with respect to time.

$$\frac{dM}{dt} = \frac{-1}{t + 43.1} \quad \checkmark$$

- (c) Calculate the decrease in mass in the 100<sup>th</sup> year.

$$M = [13.8 - \ln(100 + 43.1)] - [13.8 - \ln(99 + 43.1)] \quad \checkmark$$

$$M = -0.00701g \quad \checkmark$$

A decrease of 0.00701 g

- (d) Calculate the average decrease in the mass over 100 years.

$$\frac{M(100) - M(0)}{100} = -0.012g / \text{year} \quad \checkmark$$

**Question Three: [2, 2, 3, 2, 3 =12 marks] CA**

Kepler's Third Law states that the square of the orbit of the period of a planet,  $T$ , is proportional to the cube of its average distance from the sun,  $a$ .

- (a) State Kepler's Third Law in terms of  $T$  and  $a$ .

$T^2 = a^3$  ✓ ✓

- (b) The table below gives values of  $a$  and  $T$ , using Earth units.

Planet	Earth	Mercury	Mars	Jupiter	Saturn
$a$	1	0.3870	1.523	5.203	9.539
$T$	1	0.2410	1.881	11.86	29.46
$\ln a$	0	-0.949	0.421	1.649	2.255
$\ln T$	0	-1.423	0.632	2.473	3.383



Calculate the rows for  $\ln a$  and  $\ln T$ .

- (c) Using your CAS calculator, fit a linear regression model using the data from the table. Input the values for  $\ln a$  in List 1 and the values for  $\ln T$  in List 2. State the linear regression model.

$\ln T = 1.5 \times \ln a + 0.00027$  ✓  
✓ ✓

- (d) By stating the value of the correlation coefficient,  $r$ , explain why a linear model is appropriate.

The correlation coefficient is 1, giving a perfect linear relationship. ✓  
✓



## Mathematics Methods Unit 4

Since the vertical intercept of the answer to (c) is almost 0, we can say that  $\ln T \approx 1.5 \ln a$

- (e) Hence show using algebraic manipulation that Kepler's Third Law (as stated in part (a)) holds true from our data.

$$\ln T = \frac{3}{2} \ln a \quad \checkmark$$

$$2 \ln T = 3 \ln a \quad \checkmark$$

$$\ln T^2 = \ln a^3 \quad \checkmark$$

$$T^2 = a^3$$

### Question Four: [2, 2, 2, 3 = 9 marks] CA

Laura is starting a new fitness routine and she completes 2 sets of 5 repetitions of squats each day. Her aim is to get stronger and lift heavier each day.

Laura models her progress over  $t$  days by the function:  $f(t) = 5 + k \ln(t + 10)$ , where  $f(t)$  is the weight in kilograms of her squat each day.

- (a) Calculate the value of  $k$  if initially Laura lifts 30 kg.

$$30 = 5 + k \ln 10 \quad \checkmark$$

$$k = 10.86 \quad \checkmark$$

- (b) After 2 weeks of training, by how many kilograms has her strength increased?

$$f(14) = 5 + 10.86 \ln(24) = 39.51 \quad \checkmark$$

$$\therefore 9.51 \text{ kg} \quad \checkmark$$

- (c) Calculate the rate of change of Laura's strength with respect to time.

$$f'(t) = \frac{10.86}{t+10} \quad \checkmark$$

- (d) Determine when Laura's increase in strength is half of what it was initially.

$$f'(0) = \frac{10.86}{0+10} = 1.086 \text{ kg/day} \quad \checkmark$$

$$0.543 = \frac{10.86}{t+10} \quad \checkmark$$

$$t = 10 \quad \checkmark$$

**Question Five: [3, 2, 2 = 7 marks] CA**

The instantaneous rate of change of the number of fish over  $t$  weeks, being farmed in a fish farm can be modelled by  $P'(t) = \frac{-4970}{t+e}$  where  $P(t)$  is the population after  $t$  weeks.

- (a) If after 5 weeks there are 12 000 fish left, determine an expression for  $P(t)$ .

$$P(t) = -4970 \ln(t+e) + c \quad \checkmark$$

$$12000 = -4970 \ln(5+e) + c$$

$$c = 22156.65 \quad \checkmark$$

$$P(t) = -4970 \ln(t+e) + 22156.65 \quad \checkmark$$

- (b) Calculate the initial number of fish when the study began.

$$P(0) = 17186.65 \quad \checkmark \checkmark$$

- (c) When the decline in fish each week falls below 500, the farmer is no longer as concerned for his fish stock. During which week does this occur?

$$-500 > \frac{-4970}{t+e} \quad \checkmark$$

$$t = 7.22$$

During the 8<sup>th</sup> week.  $\checkmark$